Overarching Issues
Research suggests that mathematical understanding develops best in environments where students are encouraged to discuss mathematical concepts, student thinking is respected and valued, and sufficient time is provided for students to explore mathematical ideas (Stein, Grover, & Henningsen, 1996). Mathematical discourse allows students to explain, justify, and debate their individual techniques for solving math problems and supports the development of conceptual understanding (Trocki, Taylor, Starling, Sztajn, & Heck, 2014; National Research Council, 2001). This type of mathematical communication, sometimes referred to as math talk, bolsters student agency and ownership of their education experience while also creating an atmosphere of supportive social learning (Anderson et al, 2015). Successfully implementing mathematical discourse, however, requires teachers to drastically shift traditional teaching practices. Therefore, despite the demonstrated importance of student-centered discourse, teacher-centered instruction remains the norm in many classrooms (Franke, et al, 2009).

What is Mathematical Discourse?
When engaging in mathematical discourse, teachers introduce new mathematical concepts and provide students with problems to solve collectively. These problems are mathematical tasks with high cognitive demand, rich context, and multiple opportunities for solution, and they support a deeper conceptual understanding of mathematics (Smith & Stein, 1998; Smith & Piggott, 2007; Jackson, Shahan, Gibbons, & Cobb, 2012). Students are expected to draw from prior knowledge to develop problem-solving strategies then to explain and justify their solutions (Bruce, 2007). Teachers help facilitate discussions about student contributions by asking open-ended questions that encourage students to think critically about their mathematical approach. They also encourage other students to ask questions of their own. These questions “scaffold students’ engagement with the task, shape the nature of the classroom environment, and create opportunities for learning high-level mathematics” (Boaler & Brodie, 2004; Kazemi & Stipek, 2001; Smith, 2000; Stein, Remillard, & Smith, 2007, as cited in Franke et al, 2009). The goal of this discourse is to (1) transition from teacher as the sole questioner to teachers and students as questioners, (2) encourage students to explain their mathematical reasoning, (3) allow students’ math ideas to influence...
the direction of the lesson, and (4) promote student responsibility for learning and evaluating their progress and the progress of others (Hufferd-Ackles, Fuson, & Sherin, 2004).

**Importance of Mathematical Discourse**

The Common Core Standards for Mathematical Practice describe critical areas of expertise that students need to develop to be successful mathematical thinkers and doers. Mathematical Practice 3 (MP3) specifies that learners need to “construct viable arguments and critique the reasoning of others” in order to be mathematically proficient. This includes making predictions, analyzing mathematical situations, exploring possible solutions, creating justifications or arguments, making those arguments, being critiqued, and critiquing the arguments or solutions of others (Common Core State Standards Initiative, 2015). Thus to promote and support mathematical proficiency, it is important to develop a discourse culture and encourage consensus building within the classroom community by offering a platform for students to share their ideas, explain their logic, and assess the problem-solving strategies used by their peers (Walshaw & Anthony, 2008; Anderson et al, 2015).

**Affordances of Discourse for Strengthening Mathematical Understanding**

Mathematics discourse is seen as a vehicle for enhancing student learning. Making public conjectures and reasoning with others about mathematics allows students to develop ideas and knowledge collaboratively. As cited in Franke et al. (2009): “Describing, explaining, and justifying one’s thinking all help students internalize principles, construct specific inference rules for solving problems, become aware of misunderstandings and lack of understanding (Chi, 2000), reorganize and clarify material in their own minds, fill in gaps in understanding, internalize and acquire new strategies and knowledge, and develop new perspectives and understanding (Bargh & Schul, 1980; King, 1992; Rogoff, 1991).”

By being involved with the creation of classroom content, students assume the role of active agents, which strengthens mathematical learning overall. In addition, establishing a community wherein every student has a voice in constructing mathematical knowledge promotes equity in the classroom. For instance, when given rich mathematical tasks that encourage multiple solution strategies, can be represented in different ways, and require a variety of skills and knowledge, students with varying levels of mathematical proficiency can join the conversation and make valuable contributions (Chao, Murray, & Gutiérrez, 2014). Additionally, because mathematical discourse allows teachers to monitor and evaluate what students already know and what they need to learn (Walshaw & Anthony, 2008), teachers can use this information to inform instructional practice (Franke, Fennema, & Carpenter, 1997, as cited in Franke et al., 2009). Likewise, when teachers listen to and reflect on student contributions, they are in a better position to co-develop mathematical knowledge that connects students to the world around them (Chao et al., 2014).

**Challenges of Implementing Mathematical Discourse**

The instructional environment required for effective mathematical discourse differs greatly from the ways in which classrooms are typically organized and run. In most mathematics lessons, teachers present a math problem and the algorithm for solving it. Students then work individually to apply this algorithm to a set of similar problems, with little attention to why certain procedures are emphasized or whether they are effective strategies for all (Porter, 1989; Stodolosky, 1988, as cited in Stein, Grover, & Henningsen, 1996). When teachers ask questions, they usually require students to recall facts or rules rather than draw inferences and synthesize ideas (Graesser & Person, 1994; Hiebert & Wearne, 1993; Webb, Nemer, & Ing, 2006, as cited in Franke et al., 2009). Substantive mathematical discourse requires teachers to take on a new role in the classroom, as this approach requires less instruction and more facilitation of student ideas. Teachers are tasked with allowing students to solve the problem individually or in groups while at the same time encouraging others to listen and contribute to the
conversation. While acknowledging individual contributions, teachers must also ensure that the dialogue is moving student thinking forward toward the curricular goal. This requires sharp facilitation skills and careful attention to classroom dynamics. In the end, teachers must be able to synthesize the discussion in ways that provide students with concrete takeaways. A teacher’s willingness and ability to adopt mathematical discussion strategies, then, is dependent upon several factors, such as time, level of comfort with math content, institutional support, and available professional development opportunities (Bruce, 2007; Walsh & Anthony, 2008).

**Strategies for Successful Implementation**

One of the most important things to remember is that building a mathematical community with rich discourse is a complex process that will take time. This approach requires shifting deeply rooted instructional practices and encouraging students to assume a new, more active role in the classroom. Therefore, for many, implementing mathematical discourse is an experiment in new pedagogy and a process that is often cyclical rather than linear. For example, after a class has reached a high level of mathematical discourse, that upward trajectory is likely to fluctuate when new topics are introduced. Teacher-centered instruction often becomes more prominent until students are comfortable with new vocabulary and concepts and are able to resume more participatory roles. Students may once more begin with low-level questions and explanations and work up to higher-level discourse (Hufferd-Ackles, Fuson, & Sherin, 2004). Bearing this in mind, here is a list of recommended strategies to aide in the development of a rich mathematical community:

**Model expected behavior.** To create a classroom culture that is conducive to effective mathematical dialogue, model the behavior you expect to see and provide students with necessary instruction on group skills, shared leadership, and effective math communication (Bruce, 2007).

Recommended resources: Developing a Classroom Culture That Supports a Problem-solving Approach to Mathematics (Pennant, 2013); Ready to Learn: Creating a Positive Classroom Culture (Teaching Channel, n.d.); and Checklist for an Inclusive Classroom Community (Curriculum Services Canada, n.d.)

**Strengthen mathematics vocabulary.** For mathematical discourse to be successful, it is important to develop a shared understanding of mathematical terms. Students enter the classroom with varying levels of achievement in both written and oral language skills; some may even be classified as English language learners. Therefore, for students to adequately express their mathematical thinking and analyze the thinking of their classmates, they must have a firm grasp of the appropriate language. Allow students to develop an understanding of terms by telling stories or drawing pictures that represent their own interpretations of the words. Together, you can create a classroom chart or other tool with definitions and examples of these terms for students to refer to as needed (Pace & Oritz, 2015).

**Select rich mathematical tasks.** Pose math problems that lend themselves to multiple solution strategies which students can demonstrate in a variety of ways. (Stein, Grover, & Henningsen, 1996). Such tasks help foster classroom discourse by allowing students to “construct viable arguments and critique the reasoning of others” (Common Core State Standards Initiative, 2015). Also, pose problems with varying degrees of difficulty and relatable contexts so that all students have access to them (Math Perspectives, 2007; Piggott, 2011). Rich tasks often involve real-world scenarios that help to support students’ reasoning and communication about a particular mathematical idea (Jackson et al., 2012).

Click here for examples of rich mathematics tasks (Realizing Illinois, n.d.).

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**Rich mathematical tasks:**

- Are accessible to a wide range of learners
- Are set in relatable contexts
- Allow for learners to pose their own problems
- Allow for different methods and responses
- Offer opportunities to identify efficient solutions
- Have the potential to broaden students’ skills and/or deepen and broaden mathematical content knowledge
- Encourage creativity in applying knowledge
- Have the potential to reveal patterns or lead to generalizations or unexpected results
- Have the potential to reveal underlying principles or make connections between areas of mathematics
- Encourage collaboration and discussion
- Promote critical thinking and allow learners to develop confidence and independence

Source: Piggott, 2011
Allow students to work together in a variety of ways. This could mean working in pairs, small groups, or as a class to develop strategies and solutions. Different types of activities exercise different skills. Being just one teacher in a room full of students, however, it is challenging to observe all small group discussions. If you have the capability, let students capture their work and their thinking on iPads or other devices so that you can review it at a later time.

Ask good questions. Many teachers struggle with asking questions that reveal student thinking rather than lead students toward the correct answer. The style, substance, and quantity of questions have a significant effect on the classroom learning environment; therefore, knowing what questions to ask as well as when and how to ask them is crucial (Bofferding & Kemmerle, 2015). Asking open-ended questions encourages students to “think, analyze, criticize, and solve unfamiliar problems” (Anderson et al., 2015). Anderson et al. (2015) also define the characteristics of good questions as follows:

» Require more than remembering a fact or reproducing a skill and develop students’ higher levels of thinking
» Have more than one correct answer or approach
» Allow students to learn by answering them
» Help teachers learn about the student—providing insight into how each student thinks, what they know, and the connections they are making
» Use accessible language and offer a universal entry point for all students
» Create a sense of wonder and encourage students to construct new questions themselves

Rather than asking students to recall factual information or procedures, consider asking them to describe an alternative strategy (“Did anyone solve this problem differently?”), tell a story (“Tell a story that would match this number sequence, 20 + 35 = ?”), or generate a problem (“Can you make up a problem that is different from the previous one?”). Additionally, asking students to explain why they chose a particular strategy and how it worked or to analyze other strategies helps promote deeper mathematical understanding (Hiebert & Wearne, 1993).

Click here to see sample questions and other conversation starters (Connell, 2014).

Provide prompts. Create a poster or distribute cards with examples of appropriate questions. These can help motivate students to participate in math discourse, especially in the beginning, as well as guide the conversation toward a predetermined curricular goal (Bruce, 2007; Connell, 2014).

Click here for an example using bookmarks (Driessen, 2012).

Give students adequate time to respond. Students will likely need more time to answer questions that require higher-level thinking. Offering students adequate preparation time can result in more detailed explanations expressed with greater confidence (Bruce, 2007).

Know your students. Understanding your individual students and the classroom dynamics will help you know when to intervene and when to let the conversations continue. For instance, Walshaw and Anthony (2008) observed some teachers first monitor students’ initial participation in mathematical dialogue before deciding when to encourage participation and what type of support students need in order to join the conversation.
Take advantage of professional development opportunities.

Having a firm grasp on the subject matter will allow you to focus more on encouraging student participation. Likewise, honing your facilitation skills will allow you to focus on the content of student contributions (Bruce, 2007). According to Walshaw and Anthony (2008), “[t]eachers who were able to develop student mathematical understanding were those who had a sound base of subject knowledge. This knowledge informed their on-the-spot decision making during classroom interactions.”

Conclusion

Rich mathematical discourse offers a myriad of benefits for both students and teachers. Through mathematical discourse in the classroom, students are able to share and justify their own problem-solving strategies and analyze the reasoning of their peers as they work to solve problems collectively. In addition to supporting social learning and bolstering student agency, this approach deepens mathematical understanding overall. Furthermore, observing these classroom interactions allows teachers to evaluate what students know and what they need to learn. Because effective math communication depends on a fundamental restructuring of the classroom environment, implementing this approach is not without notable challenges. Still, the recommendations discussed in this brief can provide a solid foundation for successful implementation.

References


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